**ASSIGNMENT 1**

1. Let A={1,2,3,4,5} and R be a relation defined on A given by

R={(1,4),(2,1),(2,2),(2,3),(3,2),(4,3)(4,5),(5,1)} Find R3 .

1. If n(U)=60 , n(A)=27, n(B)=19 and n(AUB)’=31, Use Venn diagram to find: n(A∩B) and n(A-B).
2. Let A={1,2,3} and R={(1,1),(1,2),(2,3),(3,3),(1,3)} Find transitive closure R+
3. Let N={1,2,3,….} and a relation in defined in NxN as follows : (a,b) is related to (c,d) iff ad=bc, then show whether R is a equivalence relation or not.
4. If the functions f:R->R and g:R->R are defined by f(x)=x2+2 and g(x)=2x+1, then evaluate (fog)(x) , (gof)(x) and (fo(gof))(x).
5. Using laws of sets to prove that: (A∩B’)U(A’∩B)U(A’∩B’)=A’UB’.
6. If A, B, C are three sets, then show that
7. A – ( BC) = ( A- B)(A – C)
8. A(B -C) = (AB) - (AC)
9. A(B C) = (AB) (AC)
10. In a survey of 60 people it was found that 25 people read newspaper H, 26 read newspaper T, 26 newspaper I, 9 read both H and I, 11 read H and T, 8 read both T and I, and 3 read all the three newspaper. Find:
11. The number of people who read at least one of the news papers?
12. The number of people who read exactly one news paper?
13. Let X = {1,2,3,......,7 } and R = ,show that R is an equivalence relation.
14. If A is a set of positive integers and a relation R is defined on A as follows: (a,b)R(c,d) if and only if a +d = b + c for all a, b, c, d ϵ R, then prove that R is an equivalence relation.
15. Prove that the relation R = {(x, y) | x – y is an even integer for all x, y ϵ Z} is an equivalence relation.
16. List all possible functions from X={a,b,c} to Y={0,1} and indicate in each case whether the function is one to one, is onto and is one to one onto.

**SOLUTION**

**Question 1:** The relation R is defined as follows: R = {(1,4),(2,1),(2,2),(2,3),(3,2),(4,3),(4,5),(5,1)}.

To find R3 (the composition of R with itself three times), you need to perform the composition operation multiple times. Here's the calculation:

R2 = R ∘ R R2 = {(1,3),(2,2),(2,5),(3,5),(4,2),(4,4),(5,4)}

R3 = R2 ∘ R R3 = {(1,4),(2,2),(2,5),(3,4),(4,5),(4,4),(5,3)}

**Question 2:**

Given:

* n(U) = 60
* n(A) = 27
* n(B) = 19
* n(AUB)' = 31 (this means the number of elements NOT in the union of A and B)

From this, we can deduce:

* n(AUB) = n(U) - n(AUB)' = 60 - 31 = 29
* REMAINING CAN BE CALCULATED WITH THE HELP OF FORMULA.

**Question 3:** The relation R is given as R={(1,1),(1,2),(2,3),(3,3),(1,3)}.

To find the transitive closure R+, you need to find all possible pairs that can be derived from the given pairs while preserving transitivity.

Starting with the given pairs:

* (1,1), (1,2), (2,3), (3,3), (1,3)

We can derive the following pairs using transitivity:

* (1,2) (from (1,1) and (1,2))
* (1,3) (from (1,2) and (2,3))
* (1,3) (from (1,1) and (1,3))
* (1,3) (from (1,1), (1,2), and (2,3))

So, the transitive closure R+ is: R+ = {(1,1),(1,2),(1,3),(2,3),(3,3)} **Question 4:** The relation R is defined on NxN as follows: (a,b) is related to (c,d) if ad=bc. To determine whether R is an equivalence relation, we need to check for reflexivity, symmetry, and transitivity.

1. Reflexivity: For every (a,a), ad=ad holds true, so it's reflexive.
2. Symmetry: If (a,b) is related to (c,d), then ad=bc. This also implies that cd=ba, so it's symmetric.
3. Transitivity: If (a,b) is related to (c,d) and (c,d) is related to (e,f), then ad=bc and cf=de. This implies that adef=bcde, so it's transitive.

Since R satisfies all three properties (reflexivity, symmetry, and transitivity), it is an equivalence relation.

**Question 5:** Given functions f(x) = x^2 + 2 and g(x) = 2x + 1, let's evaluate the compositions:

* (f∘g)(x) = f(g(x)) = f(2x + 1) = (2x + 1)^2 + 2 = 4x^2 + 4x + 3.
* (g∘f)(x) = g(f(x)) = g(x^2 + 2) = 2(x^2 + 2) + 1 = 2x^2 + 5.
* (f∘(g∘f))(x) = f(g(f(x))) = f(g(x^2 + 2)) = f(2(x^2 + 2) + 1) = (2(x^2 + 2) + 1)^2 + 2.

**Question 6:** Using set laws to prove: (A∩B') ∪ (A'∩B) ∪ (A'∩B') = A' ∪ B'.

Proof:

1. (A∩B') ∪ (A'∩B) = (A∪A') ∩ (A∪B) ∩ (B'∪A') ∩ (B'∪B) This is by the distributive property of set operations. Since A∪A' is the universal set, and A∪B = B∪A, and B'∪B is the universal set, the above expression simplifies to: U ∩ (A∪B') ∩ U ∩ U This further simplifies to: A∪B'
2. Now, we need to combine the result from step 1 with the third term: (A'∩B'). A'∪B' = U, and U∩U = U.

Thus, (A∩B') ∪ (A'∩B) ∪ (A'∩B') = A' ∪ B'.

**Question 7:**

For this question, we have three parts to prove:

I. A - (B ∪ C) = (A - B) ∩ (A - C) To prove this, you can use the distributive law of sets and De Morgan's law.

II. A ∩ (B - C) = (A ∩ B) - (A ∩ C) This can also be proven using set manipulation and distributive laws.

III. A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) Similarly, you can use distributive laws to prove this statement.

**Question 8:** Given the information:

* Number of people surveyed = 60
* People who read H = 25
* People who read T = 26
* People who read I = 26
* People who read both H and I = 9
* People who read both H and T = 11
* People who read both T and I = 8
* People who read all three newspapers = 3

I. The number of people who read at least one newspaper: This is the union of people who read H, T, or I, minus the intersection of the pairs and adding back the intersection of all three: n(H ∪ T ∪ I) = n(H) + n(T) + n(I) - n(H ∩ T) - n(T ∩ I) - n(I ∩ H) + n(H ∩ T ∩ I) Calculate this value based on the given information.

II. The number of people who read exactly one newspaper: This is the sum of the people who read only H, only T, and only I: n(H - (T ∪ I)) + n(T - (H ∪ I)) + n(I - (H ∪ T))

**Question 10:** To prove that R is an equivalence relation, you need to show that it satisfies the properties of reflexivity, symmetry, and transitivity.

1. Reflexivity: Show that for any element 'a' in set A, (a, a) is in relation R.
2. Symmetry: Show that if (a, b) is in relation R, then (b, a) is also in relation R.
3. Transitivity: Show that if (a, b) and (b, c) are in relation R, then (a, c) is also in relation R.

**Question 11:** To prove that the relation R = {(x, y) | x – y is an even integer for all x, y ∈ Z} is an equivalence relation, you need to prove reflexivity, symmetry, and transitivity, similarly to question 10.

**Question 12:** Remember that a function is one-to-one (injective) if each element in the domain maps to a distinct element in the codomain, a function is onto (surjective) if every element in the codomain has at least one corresponding element in the domain, and a function is bijective if it is both one-to-one and onto.

Let's list all possible functions and analyze their properties:

1. Function f₁: a → 0, b → 0, c → 0
   * Not one-to-one (a, b, and c all map to the same element 0)
   * Not onto (1 is missing in the codomain)
   * Not bijective
2. Function f₂: a → 0, b → 0, c → 1
   * Not one-to-one (a and b both map to 0)
   * Onto (both elements 0 and 1 in the codomain are covered)
   * Not bijective
3. Function f₃: a → 0, b → 1, c → 0
   * Not one-to-one (a and c both map to 0)
   * Onto (both elements 0 and 1 in the codomain are covered)
   * Not bijective
4. Function f₄: a → 0, b → 1, c → 1
   * One-to-one (each element in the domain maps to a distinct element in the codomain)
   * Onto (both elements 0 and 1 in the codomain are covered)
   * Bijective
5. Function f₅: a → 1, b → 0, c → 0
   * One-to-one (each element in the domain maps to a distinct element in the codomain)
   * Onto (both elements 0 and 1 in the codomain are covered)
   * Bijective
6. Function f₆: a → 1, b → 0, c → 1
   * One-to-one (each element in the domain maps to a distinct element in the codomain)
   * Not onto (element 0 in the codomain is not covered)
   * Not bijective
7. Function f₇: a → 1, b → 1, c → 0
   * Not one-to-one (b and c both map to 1)
   * Onto (both elements 0 and 1 in the codomain are covered)
   * Not bijective
8. Function f₈: a → 1, b → 1, c → 1
   * Not one-to-one (a, b, and c all map to the same element 1)
   * Not onto (element 0 in the codomain is missing)
   * Not bijective

In summary:

* There are 8 possible functions from X to Y.
* There are 3 functions that are one-to-one: f₄, f₅, f₆.
* There are 4 functions that are onto: f₂, f₃, f₄, f₅.
* There is 1 function that is both one-to-one and onto: f₄ (bijective).

Top of Form

**SCENARIO BASED QUESTIONS:**

1. Imagine you're planning a school event. Define two sets: Set A represents students, and Set B represents available activities. Create a relation between the sets that maps students to their preferred activities.
2. Think about a real-world scenario where a function might be used. Explain the scenario, identify the input and output, and describe how the function models the relationship.
3. Describe a situation where a relation is symmetric but not reflexive. Provide a clear explanation for your example.
4. Discuss the importance of the reflexive, symmetric, and transitive properties in relations. Provide examples to illustrate each property's significance.
5. Compare and contrast one-to-one (injective) and onto (surjective) functions. Provide examples of each type of function and explain why they are classified as such.
6. Explore the concept of equivalence relations. Define an equivalence relation and give an example from everyday life. Explain how this relation satisfies the reflexive, symmetric, and transitive properties.Top of Form

**Solution**

**1**.

* Student set A: {Alice, Bob, Carol, David}
* Activity set B: {Football, Painting, Chess, Dancing}
* Relation R: {(Alice, Painting), (Bob, Dancing), (Carol, Football), (David, Chess)}

2.

* Scenario: Online Shopping
* Input: Items in the shopping cart
* Output: Total cost of the items
* Function: CostCalculator(cart) = total cost

3.

* Symmetric but not reflexive relation example: "Is sibling of"
  + Symmetric because if A is a sibling of B, then B is a sibling of A.
  + Not reflexive because an individual is not usually considered their own sibling.

4.

* Reflexive property: Ensures each element is related to itself. Example: Identity relation.
* Symmetric property: Implies if A is related to B, then B is related to A. Example: "Is a friend of."
* Transitive property: If A is related to B and B is related to C, then A is related to C. Example: "Is an ancestor of."

5.

* One-to-one (Injective) function: Each input maps to a unique output. Example: f(x) = 2x.
* Onto (Surjective) function: Every element in the codomain is mapped to by at least one element in the domain. Example: f(x) = x^2.

6.

Equivalence relation: A relation that is reflexive, symmetric, and transitive. Example: "Is congruent modulo 5."

* + Reflexive: a ≡ a (mod 5)
  + Symmetric: If a ≡ b (mod 5), then b ≡ a (mod 5)
  + Transitive: If a ≡ b (mod 5) and b ≡ c (mod 5), then a ≡ c (mod 5)

**More Scenario Based Questions**

1: Scenario: Social Media Connections Consider a scenario where users on a social media platform can connect with each other. Define a relation that represents the "is friends with" connection. Discuss whether this relation is reflexive, symmetric, and/or transitive.

2: Scenario: Course Enrollment In a university, students can enroll in courses. Define a function that maps each student to the courses they are enrolled in. Discuss whether this function can be both injective and surjective.

3: Scenario: Transportation Routes Imagine a city with multiple bus stops and several bus routes connecting them. Define a relation that represents the "can be reached by taking a single bus ride" connection. Discuss the properties of this relation in terms of sets, relations, and functions.

4: Scenario: Equivalence Relations in Programming Consider a programming scenario where you are implementing a group classification system. Define an equivalence relation that categorizes elements in the program's data structures. Explain how this equivalence relation can help optimize certain operations.

5: Scenario: Database Relationships In a database management system, tables often have relationships. Define a function that represents a relationship between two database tables. Discuss how this function can help retrieve data efficiently in queries.

6: Scenario: Graph Theory and Networks Imagine a network of interconnected devices in a smart home system. Define a relation that represents the "can communicate with" connection between devices. Explain how this relation can be modeled using graph theory concepts.

Solutions

1:

* Reflexive: Everyone is friends with themselves.
* Symmetric: If A is friends with B, then B is friends with A.
* Transitive: If A is friends with B and B is friends with C, then A is friends with C.

2:

* This function can be injective if each student is enrolled in only one course.
* This function can be surjective if each course has at least one student enrolled in it.

3:

* Reflexivity: Each stop is reachable from itself.
* Symmetry: If you can reach stop A from stop B, you can reach stop B from stop A.
* Transitivity: If you can reach stop A from stop B and stop B from stop C, you can reach stop A from stop C.

4:

* Equivalence relation: Elements are equivalent if they share a common property, like being in the same programming group.

5:

* Function: Represents relationships between tables in a database.
* This function helps in optimizing queries by joining tables efficiently.

6:

* Relation: Represents which devices can communicate directly with each other.
* Modeling using graph theory: Devices as nodes, connections as edges in a graph.